

## **SIMPLE METHOD FOR DESIGN OF MP FIR FILTERS USING REMEZ ALGORITHM AND SHARPENING TECHNIQUE**

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### **ABSTRACT:**

*Minimum-phase (MP) Finite Impulse Response (FIR) filter generally has all its zeros inside the unit circle. The group delay for MP filters is always less than group delay for the non-minimum-phase filters. Additionally, they have the minimum energy delay property, i.e. the energy of the MP system is delayed the least of all systems having the same magnitude response. In this paper we present design of a special class of MP filters, which have zeros, not only inside the unit circle, but also on the unit circle. One simple method for design of MP filters for a given specification is proposed. In the first step, the FIR linear-phase filter is designed using Remez algorithm for a given specification in which the permitted deviations in passband and stopband are increased in order to get a lesser order of the filter. In the next step all zeros in outside positions are moved to their inverse positions inside the unit circle. Finally, the sharpening technique is applied in order to reach the desired filter specification. The design is performed in Matlab. The method is illustrated with different examples.*

### **1. INTRODUCTION**

Minimum-phase (MP) filters find applications in communications, speech processing, and predictive coding, among others [1-3]. Additionally, they have the minimum energy delay property, i.e. the energy of the MP system is delayed the least of all systems having the same magnitude response [3]. In this paper is presented design of a special class of a (MP) Finite Impulse Response (FIR) filters, which have all zeros, not only inside, but also on the unit circle.

There exist two main approaches for MP filter design:

- Methods based on the low-pass (LP) FIR prototype filter design [1-2, 4].
- Methods based on complex cepstrum [3, 5].

Here is considered the method based on LP FIR filter prototype. In the first step, the prototype filter is designed, and all its zeros are found. In the second step, all zeros which are outside the unit circle are reflected to their reciprocal positions inside the unit circle. This process is called mipizing [6]. This process does not change the magnitude response of the filter [3]. In order to make mipizing more efficient, it is proposed here to design the prototype filter with the relaxed specification and apply the sharpening technique. Sharpening was introduced in [7] to simultaneously improving the pass-band and stop-band of the prototype filter, and can be only used for linear-phase filters. In the method proposed here, the simplest sharpening polynomial  $Sh\{X\}=3X^2-2X^3$ , where  $X$  is the prototype filter, is used.

The rest of the paper is organised in the following way. Section 2 describes all steps of the proposed design and it is illustrated with examples. All examples are realized in Matlab. Discussion is provided in Section 3. Finally, conclusion is presented in Section 4.

## 2. DESCRIPTION OF PROPOSED METHOD

The goal is design a minimum-phase filter  $H_m(z)$  for a given specification, which includes:

- Pass-band edge frequency in the normalized form  $\omega_p$ .
- Stop-band edge frequency in the normalized form  $\omega_s$ .
- Ripple in the pass-band  $R_p$  [dB].
- Attenuation in the stopband  $A_s$ [dB].

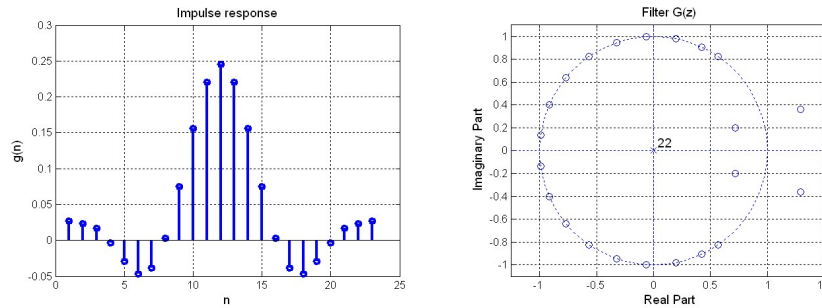
All steps of the proposed design, taking  $\omega_p=0.2$ ,  $\omega_s=0.3$ ,  $R_p=1$  dB, and  $A_s=50$  dB, are described in the continuation.

### 2.1. Design of prototype linear-phase LP FIR filter

The prototype filter  $G(z)$  is designed using well known Remez algorithm in Matlab, taking the same values  $\omega_p$  and  $\omega_s$ , as in MP filter specification. However, the pass-band ripple and stop-band attenuation are defined as:  $R_{pG}=a_1 R_p$  and  $A_{sG}=a_2 A_s$ , respectively, where  $a_1$  and  $a_2$  are positive constants,  $a_1>1$  and  $a_2<1$ . Those constants are chosen in a way to decrease the order of the filter  $G(z)$  and complete the condition that the order of the filter is an even number. In this design is taken  $a_1=1.2$  and  $a_2=3/5$ , resulting in  $R_{pG}=1.2$  dB and  $A_{sG}=30$  dB.

The designed filter has an order  $N_G=22$ .

Figure 1 shows the impulse response and pole-zero plot for the designed filter.



a. Impulse response

b. Pole-zero plot.

Figure 1: Prototype filter  $G(z)$ .

Note that the impulse response has the point of symmetry and  $N_s=12$ , and that there are two zeros outside the unit circle.

### 2.2. Mipizing

First, zeros of the filter  $G(z)$  are obtained using Matlab command `roots`. The zeros, with the absolute values more than 1, are eliminated, and the zeros, with the absolute values less than 1, are doubled. The obtained set of zeros presents zeros of the mipized filter. Next, using the Matlab instruction `poly`, the impulse response of the mipized filter, is obtained.

Figure 2 shows the impulse response and pole-zero plot of the mipized filter  $G_1(z)$ .

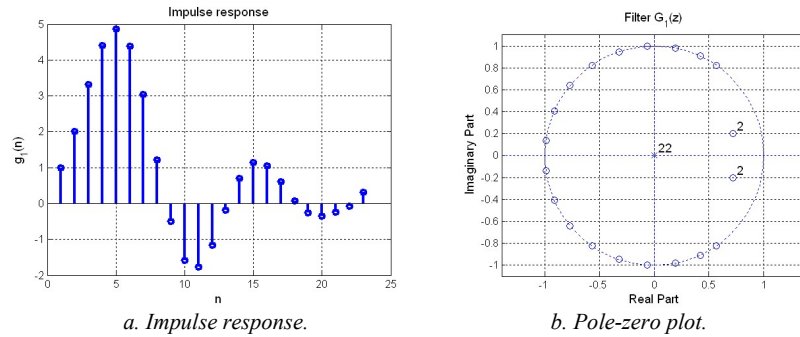


Figure 2: Mipized filter  $G_1(z)$ .

### 2.3. Sharpening

Sharpening can only be applied to the linear-phase filter [7]. To this end, we consider sharpening filter  $G(z)$ :

$$G_{sh}(z) = Sh\{G(z)\} = 3G^2(z) - 2G^3(z) = G^2(z)[3z^{-(N/2-1)} - 2G(z)] = G^2(z)B(z), \quad (1)$$

where  $B(z)$  is equal to:

$$B(z) = 3z^{-(N/2-1)} - 2G(z). \quad (2)$$

Figure 3 shows the the impulse respons  $b(n)$  and pole-zero plot.

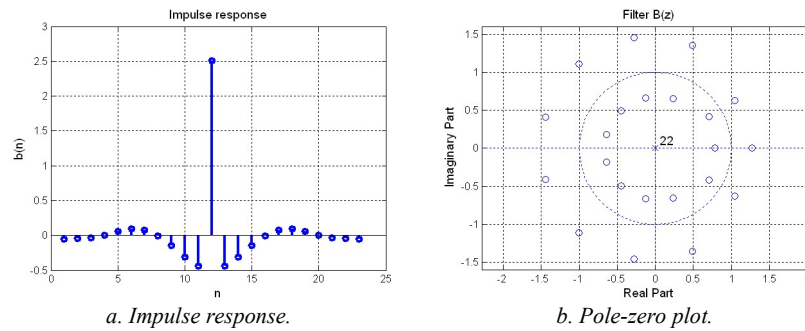


Figure 3: Impulse response and pole-zero-plot of filter  $B(z)$ .

Note an interesting feature of the positions of zeros in  $z$ -plane. There is no zero on the unit circle, and each zero, outside the unit circle, has the reciprocal pair inside the unit circle.

### 2.3. Mipizing filter $B(z)$

The zeros outside the unit circle are eliminated and zeros inside the unit circle are doubled. The corresponding mipized filter is denoted as  $G_2(z)$ . Figure 4 shows the impulse response and the pole-zero plot of the filter  $G_2(z)$ .

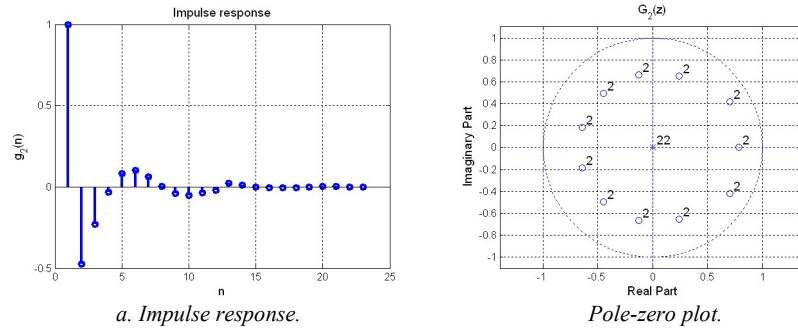


Figure 4: Mipized filter  $G_2(z)$ .

### 2.4. Final step: Design of desired MP filter $H(z)$

Using (1) we get transfer function of the MP filter  $H_m(z)$ , from the sharpened filter  $G_{sh}(z)$ , replacing the filters  $G(z)$  and  $B(z)$  with the mipized filter  $G_1(z)$  and  $G_2(z)$ , respectively.

$$H_m(z) = G_1^2(z)G_2(z). \quad (3)$$

Magnitude response of the designed filter, along with the passband zoom, is shown in Figure 5.

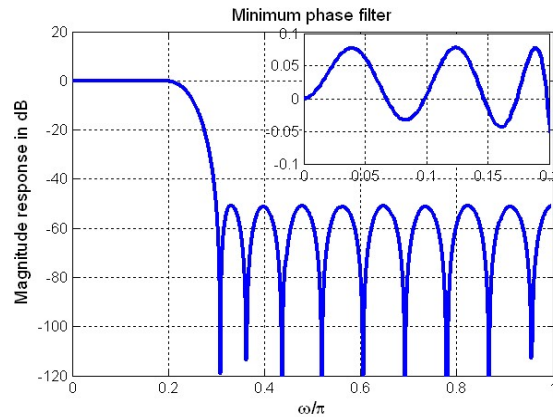


Figure 5: MP filter  $H_m(z)$ .

Figure 6 presents pole-zero-plot of the designed filter and compares the groupdelay in the passband, of the designed MP filter with the corresponding linear-phase filter.

### 3. DISCUSSION

Unlike methods in [1-2] the presented method can be applied not only for narrowband, but also for wideband MP filter design. Next example presents a wideband MP filter design.

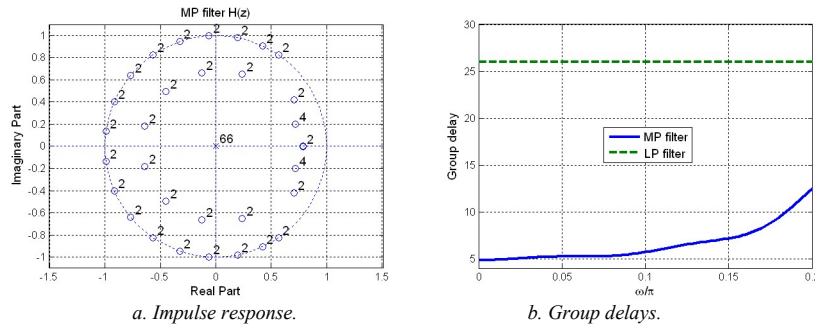


Figure 6: Pole-zero plot and group delay of MP filter  $H_m(z)$ .

**Example 1:** The specification for the MP filter is:  $\omega_p=0.7$ ,  $\omega_s=0.78$ ,  $R_p=1$  dB, and  $A_s=60$  dB. The magnitude response along with the passband zoom, of the designed filter, is shown in Figure 7 a. Figure 7 b presents pole-zero plot of the designed filter.

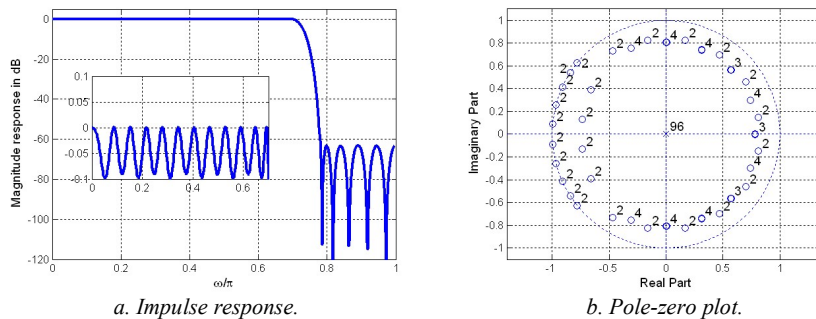


Figure 7: Designed MP filter in Example 1.

Next example shows that the proposed method can also be used for design of a narrowband MP filter. **Example 2:** The passband and stopband frequencies are 0.1 and 0.2, while the passband ripple is 0.15 dB and attenuation 70 dB. Figure 8 presents the designed filter.

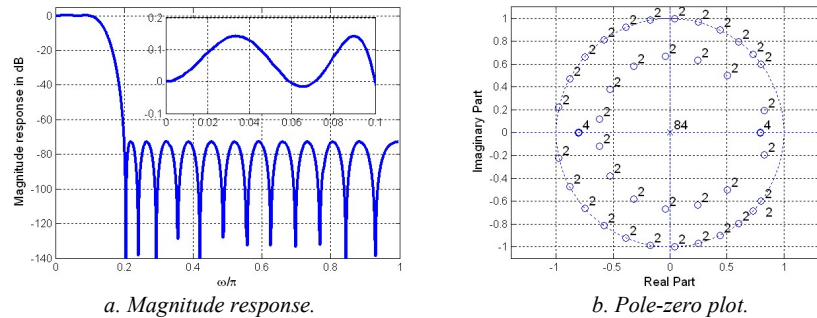


Figure 8: Designed MP filter in Example 2.

#### 4. CONCLUSION

This paper presents method for MP filter design for a given specification which includes the passband and stop band edge frequencies, passband ripple and stopband attenuation. First, a linear-phase filter is designed with the relaxed specification in order to get lower filter order. Next, the minimizing and sharpening techniques are applied to get the desired MP filter. Proposed method can be used for narrowband and wideband MP filter design.

#### 5. REFERENCES

- [1] Jovanovic Dolecek G., Dolecek, V. (2001). Method for narrowband minimum phase filter design. Electronics letters, 324-325, <https://doi.org/10.1049/el:20010185>.
- [2] Jovanovic Dolecek G., Dolecek, V. (2009) Application of Rouché's theorem for MP filter design. Applied Mathematics and Computation, 329-335. <https://doi.org/10.1016/j.amc.2009.01.058>.
- [3] Oppenheim, A. V., Schaffer, R.W. (2009). Digital Signal Processing, 3 ed., NJ: Prentice-Hall, <https://doi.org/10.1007/s11047-018-9720-z>.
- [4] Jovanovic Dolecek, G., Fernández-Vazquez, A. (2015). Sharpening minimum-phase interpolated finite impulse response filters. Computación y Sistemas, 805-815. doi: 10.13053/CyS-19-4-2075.
- [5] Pei, S. C., Lin, H. S. (2006). Minimum-phase FIR filter design using real cepstrum. IEEE Transactions on Circuits and Systems-II: Express Briefs, 1113–1117. <https://doi.org/10.1109/TCSII.2006.882193>.
- [6] Kale, I. Cain, G. D., Morling, R.C.S. (1995). Minimum-phase filter design from linear-phase startpoint via balanced model truncation. Electronics letters, 1728–1729, doi: 10.1049/el:19951228.
- [7] Kaiser, J. F., Hamming, R.W. (1977). Sharpening the response of symmetric non-recursive filter. IEEE Transactions on Acoustic, Speech, and Signal Processing, 415–422. doi:10.1109/TASSP.1977.1162980.