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## **EXPERIMENTAL DETERMINATION OF FRACTURE MECHANICS PARAMETERS IN ELASTIC-PLASTIC CONDITIONS**

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**Abstract:** *The paper describes the procedure and method of experimental determination of fracture mechanics parameters in the condition of elastic-plastic fracture mechanics, EPFM, by testing Single-edge notched bend, SENB specimen components of structural steel welded joint. A program was developed to determine the corrective value of the crack length (adjusted crack length) in the C# programming language. The results of experimental studies are presented in the form of J-R curves, and certain critical values of J integral  $J_{Ic}$  are determined.*

**Keywords:** *fracture mechanics, J integral, J-R curve, initial crack, SENB specimen.*

### **1. INTRODUCTION**

When stress, whose values exceed the yield stress limit of the material, occur in the vicinity of the crack, we often have a situation without plastic strain but rather crack growth and formation of new discontinuities. Because of these phenomenon characteristic to structural materials, it is important to note the presence of cracks in the structure, and thus the behavior of the material in the area around the crack. Hypothetically, if we were able to detect the critical cracks in the structure, it would be possible to avoid failures and disasters. As this is not possible in most cases, i.e. it is practically impossible to detect all cracks in the structure, the basic thesis that the fracture mechanics begins with is that such cracks already existed in the material, and it is necessary to define the values which will accurately describe the behavior of the material in vicinity of the crack.

Defining stress condition around the crack tip is a complex process because the stress as physical size is not defined in terms of geometric crack sizes. It is therefore necessary to define a new size that will, in addition to the static sizes of stress and strain in the material, also contain geometric characteristics of crack in its function. When the stress size around the crack tip is analyzed in classical

manner, we see that the stress increases exponentially when approaching the crack tip. We can easily observe that the crack tip becomes singular stress point. In other words, the stress size at the crack tip tends to infinity. As there is no material that can withstand infinite stress size, the crack tip is plasticized, and the size of the plastic zone largely depends upon the mechanical properties of the material [1].

Research conducted in the last 50 years for the most part had intention to confirm the basic concepts of fracture mechanics, i.e. to use theoretical knowledge when designing structures to have more enhanced resistance to crack growth. The development of fracture mechanics was based on theoretical-experimental principles, because negative experience on ships and airplanes in the middle of the last century, for the most part as a consequence had a lack of knowledge of crack growth in the structures. Prior to 1960, fracture mechanics principles were applied only on materials that are deformed according to Hook Law, and since 1948 small plastic corrections were conducted around the crack tip, with EPFM principles still being used. After 1960, non-linear materials were also examined, which led to the use of EPFM, in addition to existing theory [2].

## 2. LINEAR - ELASTIC FRACTURE MECHANICS

The crack in the body can be loaded so that there are three ways of its opening. Mechanical properties at such body can be described with stress intensity factor  $K$  which varies depending on the mentioned methods the crack opening, i.e. by defining the stress intensity factor  $K_I$ ,  $K_{II}$ ,  $K_{III}$ . Figure 1 shows the three basic models of crack opening, i.e., splitting, sliding and shear, respectively.

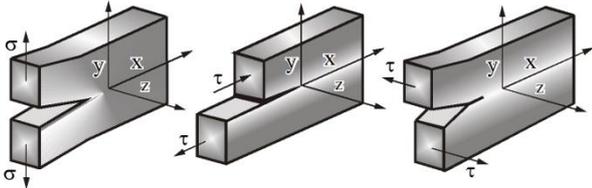


Figure 1: Crack growth models

The fundamental concept of Linear-elastic fracture mechanic (LEFM) lies in the fact that the stress field around the crack tip can be described by one parameter  $K_I$ , which represents stress intensity factor whose unit is  $N\sqrt{m}$ . This parameter is directly related to the stress intensity or the size of the crack  $a$ . It can be observed as a basic unit of fracture mechanics, by analogy with the stress unit in resistance of the material [3].

The stress intensity factor  $K_I$ , can be functionally related to stress  $\sigma$ , and the crack size  $a$ , for different forms and crack opening position on the plate, according to Figure 1. In general, stress intensity factor expressions for plate loaded with tension can be presented with the expression for all three types of crack opening:

$$K_{I,II,III} = Y\sigma\sqrt{\pi a}, \quad (1)$$

where stands that:  $Y$  – dimensionless number that depends on the geometry of the crack opening displacement shown in Figure 1.

When we consider critical values of these sizes, then the stress intensity factor exceeds the fracture toughness,  $K_{Ic}$  and represents that stress value at which unstable crack growth begins, and crack length reaches its critical size  $a_c$ . It is possible to determine the values of fracture toughness  $K_{Ic}$ , for different types of materials, which are loaded with alternating load, with a certain plate thickness, and the specific operating temperatures.

Application of LEFM is limited to a relatively small area of plastic around the crack tip. ASTM standard defines a simple condition of LEFM application that relates to experimental testing. The condition of application refers to the sizes of the specimen examining the fracture toughness  $K_{Ic}$  [4]:

$$a, B, (W - a) \geq 2,5 \left( \frac{K_I}{\sigma_{p0.2}} \right) \quad (2)$$

where stands that:  $a$  – crack length,  
 $B$  – specimen width,  
 $W$  – specimen height,

$\sigma_{p0.2}$  – yield stress

The common practice is that the width and size of the crack is expressed by one size  $b = (W - a)$ . If the condition is fulfilled (2), it provides the flat state of strain and fracture toughness becomes fundamental fracture mechanics parameter.

## 3. ELASTIC – PLASTIC FRACTURE MECHANICS

In case when the requirement of LEFM application is not fulfilled, or if the plastic zone around the crack tip is insignificant, the obtained fracture toughness parameter  $K_{Ic}$  is not adequate to define the boundary of stable crack growth, and therefore it is necessary to conduct further analysis, and to include non-linear material behavior. High value of fracture toughness and low boundary of the yield strength is characteristic to material that is necessary to treat with EPFM. Such materials are often used in the manufacturing of pressure vessels. In case of EPFM primary parameters are J integral and crack tip opening displacement  $\delta$  (CTOD). The critical values of these parameters present the value of fracture toughness, even for relatively large plastic zone around the crack tip. There are limitations to the size of the plastic zone in EPFM applications, but they are significantly less restrictive in relation to the LEFM [3].

### 3.1. J Integral

Analogously, as in the case of LEFM, it is possible to define the non-linear strain energy  $J$  that is released during flat plate load. In the same manner, as in the case of strain energy  $G$  in LEFM, non-linear strain energy  $J$  can be used as a fracture criterion, so that for a given material at a critical value  $J_c$  unstable crack growth occurs.

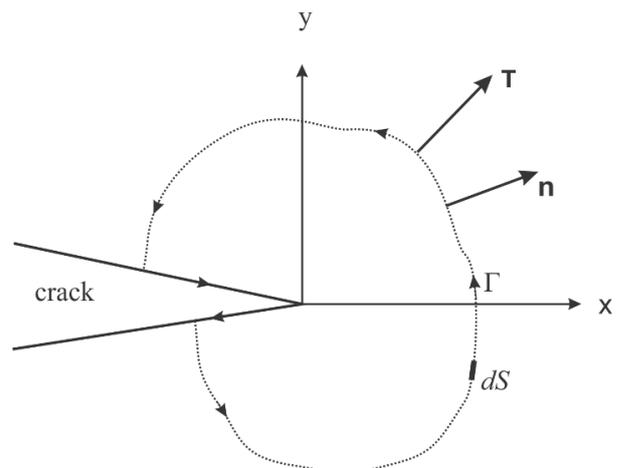


Figure 2: 2D cracked body with contour G in counterclockwise direction

Integral equation proposed by James R. Rice (1968) can calculate the strain energy of the body that contains the crack. This integral, named after the scientist who first defined it, is known as the  $J$  integral.  $J$  Integral can also predict when the unstable crack growth will occur [5].

For each homogeneous and isotropic body that shows the nonlinearity of its elastic properties in the balanced state, it is possible to determine the definite integral  $J$ , defined by a closed line whose value is equal to zero. In other words, it is possible to define the contour integral, which does not depend on the closed line path, and its value is always equal to zero.

If the crack exists in the body, and based on the previous discussion, it is possible to define a closed line in the case of crack. If the starting point of a closed contour begins at the bottom of the crack and ends at the upper surface of the crack (Fig. 2) we can determine:

$$J = \int_S \left( W dy - T_i \frac{\partial u_i}{\partial x} dS \right). \quad (3)$$

The above expression (3) is the general form of Rice  $J$  integral, and provides a starting point for the determination of fracture mechanics parameters in elastic-plastic conditions.

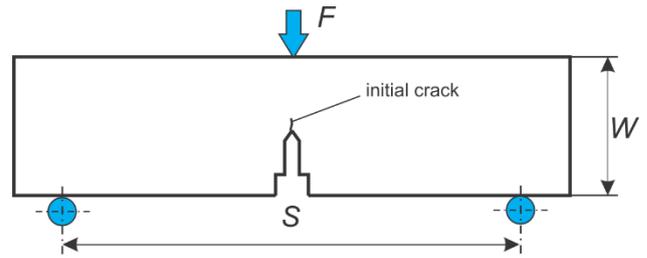
#### 4. EXPERIMENTAL RESEARCH

Experimental determination of fracture mechanics parameters consists of a set of experimental methods for determining fracture toughness  $K_{IC}$ , by applying the principles of LEFM to real structures. Structures with these properties are built from high strength materials that are not in mass use due to the high cost. On the other hand, construction materials that are massively used are not suitable for the application of LEFM primarily because of their mechanical properties, so we use the EPFM. In such conditions, the fracture toughness can be calculated using one or more parameters such as:  $J$  contour integral, or the crack tip opening displacement  $\delta$ .

These parameters ( $J_{IC}, \delta_{IC}$ ) are suitable for the analysis in EPFM which makes them very important in the calculation of the structural safety. Experimental methods used to determine these parameters are defined by the American ASTM, British BS and ISO standards. There are several standards [6, 7, 8] which determine procedures for the experimental determination of fracture mechanics parameters, primarily fracture toughness at plane strain  $K_{IC}$ , critical  $J$  integral  $J_{IC}$ , and critical crack tip opening displacement  $\delta_{IC}$  (CTOD). The aforementioned standards, among other, prescribe the form and dimensions of the specimens that are used to determine these parameters. For the experimental determination of fracture mechanics parameters SENB specimens were used. Standard geometry of SENB specimens has a rectangular cross-section, with cross-section size  $B \times 2B$ , where:  $W = 2B$ ,  $B$  - specimen thickness,  $W$  - specimen width. An example of one such specimen is given in Figure 3.

Prior to the determination of fracture toughness, the initial fatigue crack in three-point bending with alternating load is formed for each specimen. The size of the crack makes the total length of machine etched notch and initially formed fatigue crack (Figure 3), whereas its standard size is recommended in the ratio of 0.45-0.70  $W$ . The initial formation of the crack should be strictly controlled, because it directly affects the fracture toughness [7]. Load

which affects the specimen is also determined by the standard, and it is recommended to be placed in interval - 1 to 0.1 maximum load.



**Figure3:** Single-edge notched bend (SENB) specimen

According to ASTM 1820 [4], the maximum load of the specimen during the formation of the initial fatigue crack is given in the expression:

$$F = \frac{0.5 B b_0^2 \sigma_Y}{S} \quad (4)$$

It stands that:  $B$ —specimen thickness,  
 $b_0$ —remaining length of the ligament,  
 $\sigma_Y$ — effective yield strength,  
 $S$ —distance between supports.

After the formation of the initial crack it is possible to conduct a main test for the determination of fracture mechanics parameters. The main test was conducted using single specimen partial unloading testing, i.e., single specimen compliance method. The whole process of this method is defined by the standard ASTM 1820 [4]. The aim of this method is to register the size of the crack growth,  $\Delta a$ , which occurs during the test, over the size of the crack tip opening displacement which is registered by the special COD extensometer installed in the specimen. The load is introduced so that the maximum value is reached after no more than 10 minutes, whereas unloading should not be higher than half the current load. The load is introduced with occasional unloading to the moment of large plastic strain or fracture of the specimen. Termination of the experiment can be achieved when leaving measurement range of the extensometer, or until it reaches a sufficient number of cycles for the construction of J-R curve. In cases where the specimen is not broken after the experiment, it is broken with force in order to continue the process of measuring the crack length. Upon the completion of the test the position of the crack caused by bending of the specimen is marked. Marking the position of the crack on the specimen is conducted by fatigue or thermal treatment. After marking, the specimen is broken in order to calculate the initial  $a_0$ , or final crack length  $a_f$ . The experimental results, which are collected in order to determine the fracture mechanics parameters, are obtained in the form of load - crack tip opening ( $F - v$ ), or yield curve. Based on the yield curve it is possible to determine the stress intensity factor for each cycle, given in the expression:

$$K_i = \left[ \frac{P_i S}{\sqrt{B B_N W^3}} \right] f \left( \frac{a_i}{W} \right) \quad (5)$$

Obtained values of the stress intensity factor have shown that in our case the requirement for the specimen thickness was not fulfilled:

$$B \geq 2,5 \left( \frac{K_i}{\sigma_{p0,2}} \right). \quad (6)$$

Since the thickness of the tube (6) is inadequate, meaning that the plastic zone around the crack tip is significantly large, it is therefore necessary to apply the principles of EPFM. In this case it is necessary to proceed with the calculation and determine the parameters of EPFM, and  $J$  integral for each unloading cycle. Several standards prescribe procedures for determining the  $J$  integral [4, 9, 10] of which the ASTM E1820-01 [4] defines the procedures and guidelines for the calculation of fracture mechanics parameters such as  $K$ ,  $J$ ,  $\delta$ .

Based on the calculated values of the stress intensity factor of each cycle it is possible to reach values of  $J$  integral for each cycle through the expression:

$$J = J_{el} + J_{pl}, \quad (7)$$

where  $J_{el}$  and  $J_{pl}$  present elastic, i.e. plastic component of  $J$  integrals.

$J - R$  Structure begins by applying the maximum values of  $J$  integral  $J_{max}$  and crack growth  $\Delta a_{max}$ . Cycles of failure are measured in equal values of crack length growth of  $0.005W$ .

For methods with alternate failure, it is possible that the calculated crack length is less than the initial value, so it is necessary to make an adjustment (rectification) of the crack length in order to obtain a realistic  $J - R$  curve. The process of rectification of the crack length applies to all cycles until reaching the maximum load. By using such a defined set of data from  $J_i$  and  $a_i$ ,  $a_{oq}$  is calculated using the following expression [4]:

$$a = a_{oq} + \frac{1}{2\sigma_Y} J + B J^2 + C J^2. \quad (8)$$

Unknown sizes  $a_{oq}$ ,  $B$  and  $C$  in the expression are calculated by the least square method, and the expression (8) is transformed into the matrix equation. The paper implemented algorithm for calculating rectified crack size  $a_{oq}$  in the C # programming language [1].

In accordance with the prescribed procedure, at least 8 points are necessary to define the values  $a_{oq}$ . At least three of the aforementioned 8 points must be in the interval  $0.4J_Q - J_Q$ . The correlation coefficient for the purpose of calculating the coefficients  $B$  and  $C$  should be greater than 0.96, and the difference between  $a_0$  and  $a_{oq}$  less than  $0.01W$ . Based on these limitations we can obtain successive growth  $a_i$  from the expression:

$$\Delta a_i = a_i - a_{oq} \quad (9)$$

The values of  $J_i$  are obtained through the expression (7), with the rectified value of the crack length. Once you have determined the value of  $J$  integral and crack growth

for each cycle, they can be applied in the diagram  $J - \Delta a$ . After all values are entered in accordance with the procedure of the J-R structure it is possible to determine the critical J integral  $J_{Ic}$ .

#### 4.1. The Research results

According to the described procedure of the experimental determination of fracture mechanics parameters, and experimental determination of fracture toughness in plane strain, the results are given in the diagram in the yield curve (*Unloading Compliance Toughness Test*) and J-R curve with which critical value of  $J_{Ic}$  is determined [1].

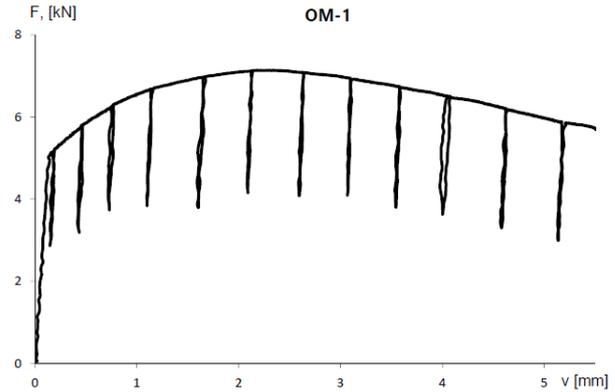


Figure 4: Unloading compliance toughness test record

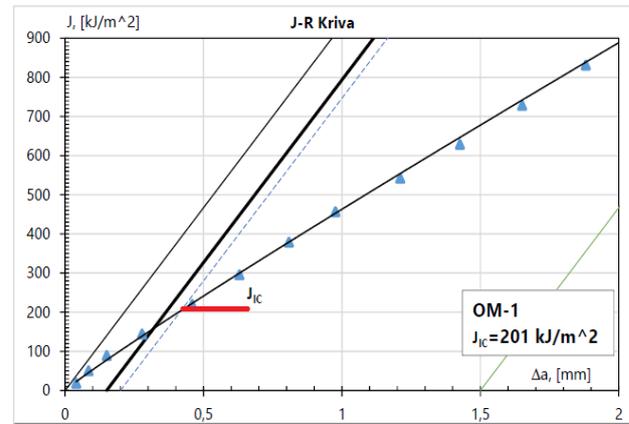


Figure 5: Calculated J-R curve with critical value of  $J_{Ic}$

Figure 4 shows the obtained yield curve at which 12 alternating cycles of loading and unloading were conducted. Previously described procedure for determining parameters for the J-R structure was conducted for 12 cycles of unloading.

The critical value of J integral was determined by J-R structure curve [1]. The value of the critical J integral is  $J_{Ic} = 201 \frac{kJ}{m^2}$  for the tested steel mark S 355 J2 per European standard EN. Specimen sizes used for this experiment are:  $S = 80$  mm,  $W = 20$  mm,  $a_0 = 10$  mm.

## 5. CONCLUSION

This paper presents experimental research regarding the experimental determination of fracture mechanics parameters in elastic-plastic conditions. The experimental part describes the procedure for obtaining J-R curve and the critical value of J integral  $J_{Ic}$ . Fracture mechanics parameters result in a functional relationship of structure stress state with its geometric properties, and the crack size. The formation of cracks and its initial stable growth still does not endanger the structure, if proper analysis is conducted and the parameters of fracture mechanics are determined. The problem occurs when the crack state in the structure changes and when the crack turns from stable to unstable growth at which fracture occurs. The determination of critical values of the stress intensity factor parameters  $K_{Ic}$  and/or J integral defines the boundary and prescribes the conditions for the safe exploitation of the structure.

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